Demand Response Control for the Inverter Air Conditioners Based on Hierarchical Nonlinear Model Predictive Control for Plug-And-Play

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ABSTRACT

Developing the demand response control for the air conditioners of residential buildings has been proven to be a highly effective strategy in assisting grid supply-demand balance and facilitating the integration of renewable energy to decarbonize the energy system. Model predictive control (MPC) has strong capabilities for unlocking the flexibility of residential buildings to realize DR by responding to electricity prices. However, the high computational requirements and complex control system integration processes make the application of MPC a significant challenge. A hierarchical nonlinear MPC (HNLMPC) is developed to realize grid-responsive control for residential inverter ACs by responding to real-time electricity price signals. The controller consists of three parts: the upper-level supervisor MPC, the lower-level optimal PID controller, and the signal converter. The indoor air temperature is selected as the optimized setpoint sequence passed from the upper level to the lower level. It could utilize cloud-based infrastructure or the Internet of Things, which means the operation not be limited by the local computing power. To enable the proposed MPC framework to perform precise demand-responsive AC control and indoor environment optimization, a nonlinear prediction model is developed considering the dynamic performances of the inverter air conditioner and the coupled thermal response of an air-conditioned room. As a result, HNLMPC enables plug-and-play capability for practical applications, reducing the dependency on local computing power, maintaining the performances, and improving the robustness. Compared to basic rule-based control, HNLMPC reduces peak-hour energy consumption by 31.6% and total electricity costs by 14.3% over the entire cooling season. Compared with centralized MPC, the HNLMPC has a lower demand for computing power.

1. INTRODUCTION

Unlocking the demand response (DR) potential in the building sector is a vital aspect to promote the balance between electricity supply and demand and facilitate the integration of renewable energy sources(*Tang et al., 2019*), assisting in the low-carbon transformation of the energy system. Among building energy consumption, space cooling has emerged as the fastest-growing segment, representing 16% of global electricity demand. The air conditioners (AC) of residential buildings have a significant share of electricity consumption and serve as a major source of peak electricity demand in summer, which has demonstrated immense DR potential. MPC is gaining popularity in the DR control of building sectors (*Zhang et al., 2022*), which enables it to handle uncertainties from both the demand and supply sides, as well as the dynamics of the building, cooling system, and their coupling (*Drgona et al., 2020*). Considering the advantages of MPC in terms of control robustness and optimization, this method is preferable for addressing optimal DR control challenges in residential ACs. However, there are still several major research gaps that need to be filled to improve optimization performance and easier implementation that is still lacking in the previous literature. (1) It is difficult for the developed centralized MPC (CMPC) to be integrated into residential ACs and operated efficiently and safely. The CMPC directly controls the operation states

of each modulatable component (such as frequency of compressor, opening of the expansion valve, speed of the fans, etc) of the ACs to keep thermal comfort. Therefore, when integrating CMPC, the original internal logic of the ACs needs to be completely overwritten, which seems impossible for the massive used ACs; The limited local computing power of ACs makes it difficult to achieve efficient MPC operation (*Jamshidi et al., 1987*).
Hierarchical MPC (HMPC) offers a systematic and flexible approach to control systems, enhancing their performance, robustness, and adaptability in complex and dynamic environments, in which the upper layer that is supervisory MPC (SMPC) computes optimal set points to realize the optimization control objective while the lower layers focus on set points tracking, which could be any kind of control method (*Rastegarpour et al., 2021*). This makes HMPC more feasible for being integrated to realize plug-and-play. HMPC can be combined with smart home technology, enabling communication between the different levels through cloud-based infrastructure or the Internet of Things (IoT)(*Drgona et al., 2020*). The AC equipped with an embedded PID controller, will then track the desired setpoints. Therefore, if the temperature setpoint is used as the output sequence of the upper layer, it can achieve better plug-and-play functionality and always satisfy comfort constraints.
This paper develops HMPC with the nonlinear model (HNLMPC) for inverter ACs of residential buildings under dynamic electricity to realize DR and keep thermal comfort, in which the upper layer is a nonlinear SMPC computing optimal indoor-air-temperature setpoint and the lower layer is an optimal PID controller focusing on fast setpoint tracking from the upper MPC. A comprehensive dynamic nonlinear model integrating the dynamic performances of inverter AC under weather and building load is developed to achieve continuously variable frequency optimization operation of the cooling system leading to improved energy efficiency and comfort. Simulation case studies based on a single-family home with inverter ACs are conducted to demonstrate the applicability of the proposed HNLMPC framework in comparison with HLMPC to explore the DR efficiency and performance sensitivity to model accuracy in building the developed MPC.

2. Development of the framework of HNLMPC

2.1 Overview of the HNLMPC

The structure of the HMPC of the residential building with an inverter AC is shown in Figure 1. It consists of three main components: (1) upper optimal controller (SMPC) that calculates the optimal indoor air temperature set point as the referenced value using the prediction models of building and inverter AC and optimizer under different disturbances and constraints to realize DR and keep thermal comfort, (2) lower controller (PID/PI) that gives operating states of the inverter AC to make indoor-air temperature reach to the reference value and (3) signal exchanger that realizes the signal exchange between slow (upper layer) and fast (lower layer) dynamic layers.

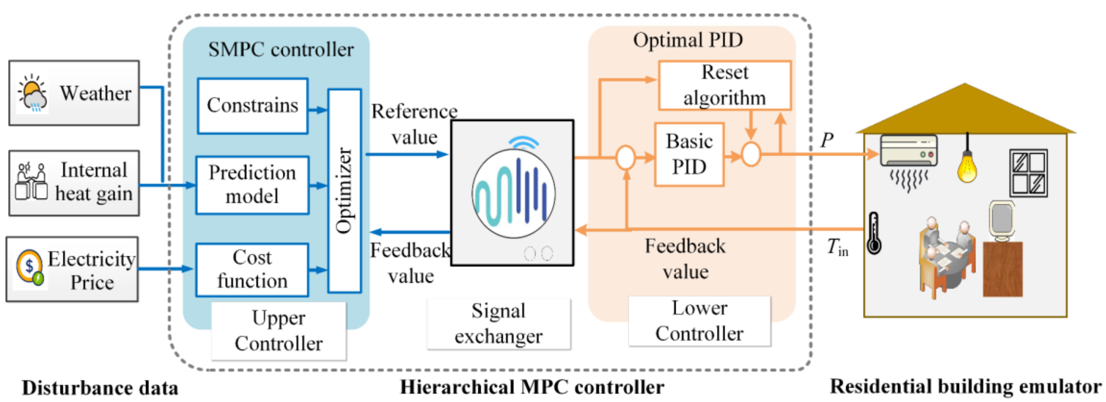

Figure 1: Schematic diagram of the hierarchical MPC framework for residential buildings with inverter ACs considering the dynamic electricity price.

For the upper controller, it considers the dynamic performance of the inverter AC and the coupling between the inverter AC and the building. First, it employs nonlinear prediction models and optimizer to obtain the optimal control outputs under the thermal comfort and power of inverter AC constraints, and then the optimal indoor air temperature setpoint (T_{ref}) of next time would be calculated and transmitted to the lower controller, as depicted in Figure 2.

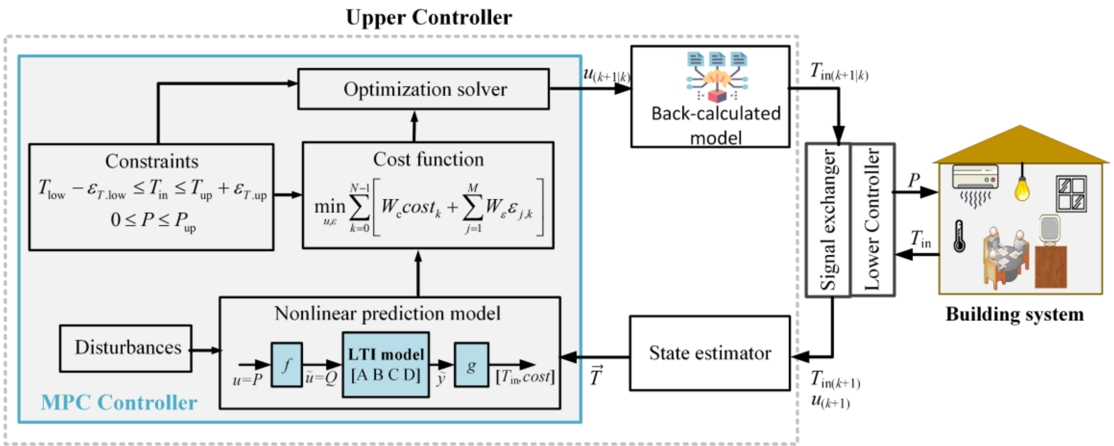

Figure 2: Schematic diagram of the computing process and variable transmission of the upper controller.

2.1.1 Prediction model of SMPC

As mentioned earlier, the relationship between the inputs and outputs of the prediction model is described in Equation. (1). To ensure the accuracy and computational efficiency of prediction, a nonlinear model is constructed, which includes three main components: (1) nonlinear static performance of Inverter AC considering the non-linear behavior of the power demand of the inverter AC system to unlock the flexibility of equipment, (2) dynamic linear resistance-capacitance building model based on the state-space formulation that predicts indoor air temperature after receiving cooling capacity from inverter AC model (*Bueno et al., 2012*) and (3) linear output model to calculate electricity cost, as shown by Figure 2.

$$\left[T_{in}, Cost\right] = f(D_{load}, P, EP) \qquad (1)$$

Where T is the temperature; *Cost* denotes the electricity cost; D_{load} is the disturbances affecting the cooling/heating load of buildings; P and EP present power of inverter AC and electricity price, respectively.
The steady-state performances of the inverter AC is selected, which is suitable for the MPC design (*He et al., 1995*). The flexibility of inverter AC is unlocked by introducing the nonlinear model shown by Equation. (2) (*Gayeski et al., 2012*).

$$Q_{cool} = P \times \left[C_1 + C_2 T_o + C_3 P + C_4 T_o^{\,2} + C_5 P^2 + C_6 T_o P\right] \; (2)$$

where C1-C6 are the coefficients derived from the inverter AC experiments; P denotes the power of the inverter AC.

2.1.2 Optimization of SMPC

The upper MPC utilizes optimization techniques to optimize the power of the inverter AC, aiming to achieve cost savings while maintaining the indoor air temperature within the thermal comfort range shown in Equation. (3) in a prediction horizon, N (*Bemporad et al., 2017*). The temperature range of thermal comfort is shown in Equation. (4), and the power range of the inverter AC is shown in Equation. (5). Because of the nonlinear performance inverter AC, the MPC belongs to a nonlinear controller that solves a nonlinear programming problem using sequential quadratic programming, which requires Optimization Toolbox™ software.

$$J = \min\left[w_c \sum_{k=0}^{N-1} cost(t+k\,|\,t) + w_\varepsilon(\varepsilon_{T.up} + \varepsilon_{T.low} + \varepsilon_{P.up}) \right] (3)$$

$$T_{low} - \varepsilon_{T.low} \le T_{in} \le T_{up} + \varepsilon_{T.up} \; (4)$$

$$0 \le P \le P_{\mathrm{up}} + \varepsilon_{P.\mathrm{up}} \qquad (5)$$

where w and ε are the penalty weighting factor and slack variable, respectively; N refers to the number of prediction horizons; the subscripts T, P, c, and ε represent the temperature, power of the inverter AC, cost, and slack variable. the subscripts low and up denote the lower and upper indoor air temperature limits, respectively; the subscripts t is the time, and (t+k|t) denote that the variable of at time t+k is predicted at time t.

2.2 Description of signal exchanger

The dynamic speeds of the upper and lower layers in the HNLMPC system are different, which necessitates the use of a signal exchanger to handle and transmit the signals between the layers. The relationship between the variables from the two layers is depicted in Figure 3. In the upper layer, the main control objective is to compute the optimal conditions for a performance index representing economic and environmental criteria over a long-term predicted horizon H_{up}, with a sampling period $\triangle\tau_{\mathrm{up}}$. Since the optimization time is significantly shorter than the sampling time, it can be ignored. At time k, the upper controller operates and calculates a series of optimal setpoints for the next N_{up} time intervals. However, only the first optimal setpoint is transmitted to the lower layer as the setpoint for the interval [k, k+1]. In the lower layer, the sampling time $\triangle\tau_{\mathrm{low}}$ must be smaller than $\triangle\tau_{\mathrm{up}}$ to ensure that the indoor air temperature approaches the setpoint before time k+1 arrives.

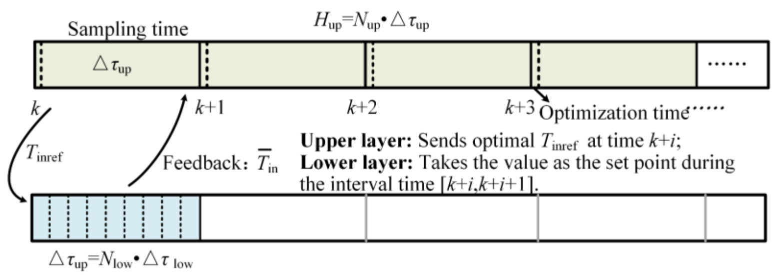

Figure 3: The simplified schematic for signal exchange between the two control layers

3. Implementation of HNLMPC

3.1 Introduction of room with inverter AC

A prototype room in a residential building was designed as the controlled object for the study, situated in Guangzhou city with a hot climate. The residential room (L = 8 m, W = 6 m, H = 3 m) has one east-facing exterior wall (6 m × 3 m), one south-facing exterior wall (8 m × 3 m) and one west-facing exterior wall (6 m × 3 m). The model assumes the partition walls between the controlled room and the adjacent room to be adiabatic, considering the temperature differences between them to be negligible. The properties of the room's envelope and internal thermal mass are summarized in Table 1. The room is cooled by a split-type inverter AC with a rated cooling capacity of 2.5 kW. The power range of the inverter AC is specified as 50–1,400 W.

Table 1. Properties of the envelope and the internal thermal mass

Description	Parameter		Value
External wall	Thermal resistance (m^2•K/W)		0.30
	Convection heat transfer(W /(m^2•K))	External surface	20
		Internal surface	9
	Conversion coefficients for solar		0.3
	Heat capacity(J/kg•K)		875
Airflow rate	Fresh air changes per hour (1/h)		0.5
Window	Heat transfer coefficient(W /m^2•K)		3
	Window wall ratio		0.13

Thermal mass	Heat capacity(J/kg•K)	1,930
	Convection heat transfer(W /m^2•K)	9
	Conversion coefficients for solar	0.05

3.2 Perameters setting of HNLMPC

When SMPC is applied to small thermal inertia rooms, the prediction horizon should be short, as 0.5–4 h (*Hu et al., 2019, Yang, et al., 2018*). The investigated room is built with light thermal mass and has small thermal inertia. Consequently, the sampling time ($\triangle t_{up}$) was set to 30 min for SMPC to respond to the dynamics of the room and ensure the stable operation of the system, during which the local controller can realize the optimal set-point tracking. The prediction horizon (H_{up}) is equal to 12h by considering the computation time cost and optimization (N_{up}=24). For the local controller, the sampling time ($\triangle t_{low}$) was 30 seconds to make the indoor air temperature approach the set point during the $\triangle t_{up}$.

3.3 Preparation of the disturbances

During the simulation process, it is necessary to obtain weather data at least one prediction horizon in advance, either through a prediction model or from an observatory. In this study, the focus is on evaluating the performance of the optimal control method, so it is assumed that the weather conditions are known in advance. The weather conditions during the cooling season in Guangzhou city are selected to investigate the performance of HNLMPC, assuming an accurate prediction of the weather data. Figure 4(a) illustrates some typical daily weather patterns. Price-based DR is well-suited for large-scale users with small capacities due to its flexibility and the end users can voluntarily adjust their consumption patterns in response to dynamic electricity prices. Real-time pricing (RTP) models offer the ability to modulate electricity prices in minutes, allowing for effective responses to fluctuations in the electricity market. RTP holds significant potential for real-time DR. In this simulation, a typical RTP at 15-minute intervals is obtained from a mature real-time electricity price market. The peak hours are from 15:00 to 18:00. The daily standard deviation (STD) of RTP for the periods of June 14th to 18th is shown in Figure 4(b). The occupancy is simplified and pre-specified. The daily occupancy pattern is divided into two parts: the rest time (from 24:00 to 8:00) and the activity time (from 8:00 to 24:00). During the rest time, the indoor thermal comfort range is set to be between 22 ℃ and 26 ℃. When the occupants are active, the indoor thermal comfort range is narrowed to between 22 ℃ and 24 ℃. The internal thermal load primarily comes from domestic appliances and the occupants themselves. To simplify the simulation, the internal load is preset as a constant value of 400 W throughout the day.

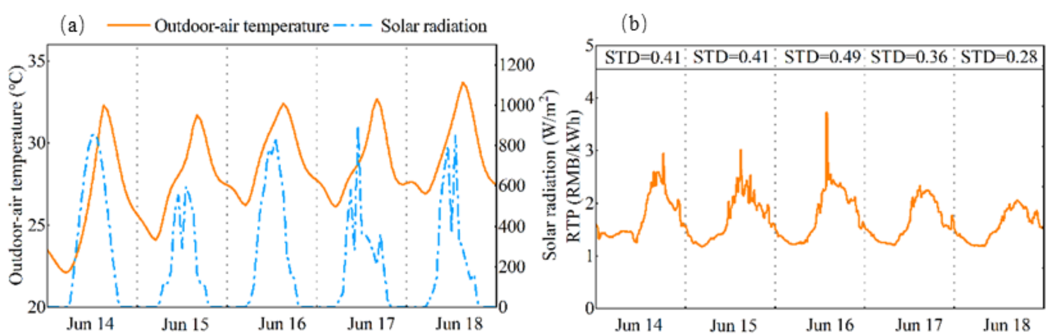

Figure 4: (a)Weather conditions and RTP for typical days from June 14th to June 18th

3.4 Control strategies

To evaluate the performance of HNLMPC, it is compared with several control methods that have demonstrated good performance. These include hierarchical controllers, whose upper-layer controllers are rule-based control (RBC) and HLMPC methods that do not consider the dynamic performance of the inverter AC, whose coefficient of performance (COP) is constant. The control methods of the lower layer also are the optimal PID controller. Central nonlinear MPC (CNLMPC) is also constructed to analyze the performances of HNLMPC, which controls the power of inverter AC directly and does not have a lower controller. The sampling time of CMPC is 180 seconds, which is the same as that of the lower layer of hierarchical controllers. Table 2 gives the descriptions of the different control methods.

Table 2. Different control methods for inverter AC.

	Upper controller			Lower controller
	Prediction model	Comfort range	Indoor temperature setpoint	Control methods
HNLMPC	Nonlinear	[22, 24], [22,26]	-	Optimal PID
Basic R-b	-	-	24, 23	PID
HLMPC	Linear	[22, 24], [22,26]	-	PID
CNLMPC	Nonlinear	[22, 24], [22,26]	-	-

4. Results

The emulator for testing the control strategies comprises two components: the inverter AC and the tested room. For the room model, Simscape is utilized to construct a discrete dynamic model that captures the nonlinear dynamics and energy consumption behavior of the building in real-time, considering various impact factors such as weather conditions and occupancy. This model serves as a testbed for evaluating different control strategies for buildings. The physical component model with Simscape is built based on physical connections that directly integrate with block diagrams, as shown in **Figure 5**. For nonlineaer AC model, it is built using Simulnk.

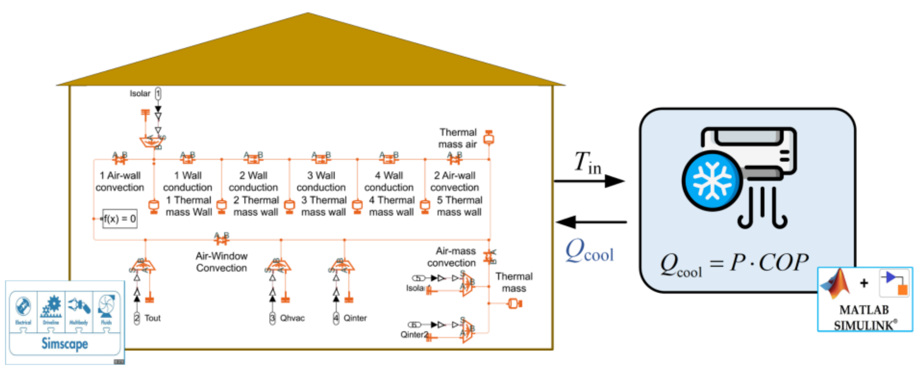

Figure 5: The tested building model in the Simscape environment

4.1 Performance comparison between HNLMPC and HLMPC

In this section, we conduct a detailed analysis of the performance improvement of the HMPC controller when considering the coupling between building and system dynamic performance using the nonlinear prediction model. For comparison, the HMPC controllers with different constant *COP* values are selected as the comparative control methods. The static *COP*s used are 3.5 (minimum *COP*), 10 (maximum *COP*), and 5 (average *COP*), which are obtained from the cooling-season control results of the basic RBC method. Figure 6 demonstrates the electrical load-shifting capability of the different HMPCs for typical days and the entire cooling season. The 1-hour moving average power depicts the charging and discharging processes of the building's thermal mass. HNLMPC gradually increases the precooling rate during the night, reducing the peak load caused by the decrease of room temperature setpoint. It continues to provide significant cooling capacity to the rooms during the second peak period, effectively reducing high electricity costs resulting from peak electricity prices and high loads between 15:00-18:00. On the other hand, other HLMPCs resulted in either over-precooling, under-precooling, or inappropriate precooling timings. **Figure 7** compares the system performances of different HMPC control methods with the Basic R-b controller for typical days and the entire cooling season. For typical days, compared with the Basic R-b, HNLMPC, HLMPC(*COP*=3.5), HLMPC(*COP*=5), and HLMPC(*COP*=10) achieved reductions in peak-hour energy of 35.4%, 30.8%, 30.7%, and 32.0%, and in total electricity cost of 20.4%, 10.4%,16.6%, and 17.0%, as shown in **Figure 7** (a). For the entire cooling season, all controllers achieved substantial peak-hour energy savings. The total cost savings for HNLMPC, HLMPC(*COP*=3.5), HLMPC(*COP*=5), and HLMPC (*COP*=10) are 14.3%, 5.1%, 10.8%, and 12.5%, respectively, as shown in **Figure 7** (b). It is important to note that incorrect *COP* values can significantly degrade performance.

Due to the hierarchical control method adopted in this study, the HMPC had strong robustness even when there were significant deviations in the performance prediction of inverter AC in the prediction model. If the dynamic performance of inverter AC affected by weather and building load variations could be considered in the prediction model, the HMPC method would have better DR performances.

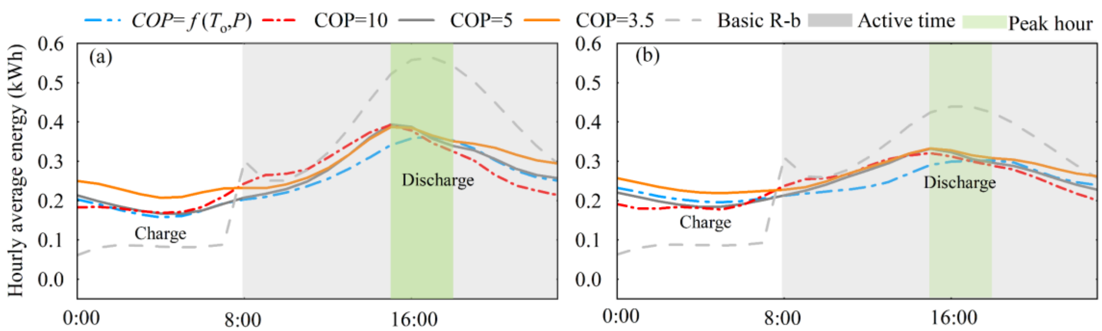

Figure 6: Comparison of daily profiles of total 1-h moving average inverter AC power between HNLMPC and HLMPC with different *COP* values for (a) typical days June 12th-18th and (b) cooling season.

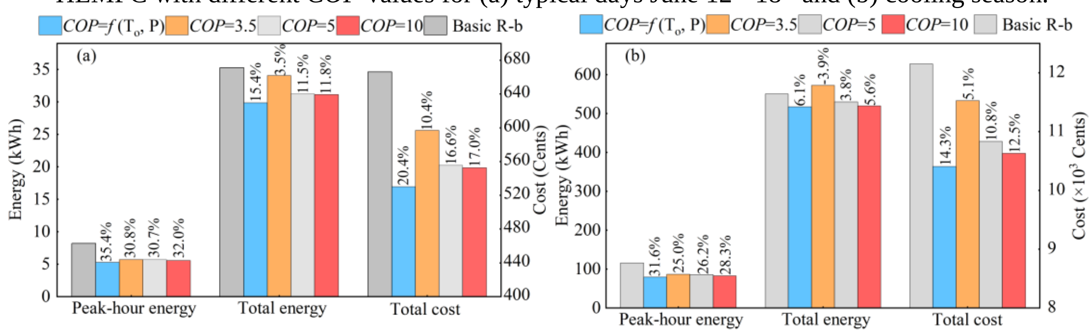

Figure 7: System performance results of the HNLMPC and HLMPC with different COP values for (a) typical days June 14th to June 18th and (b) cooling season

4.2 Performance comparison between HNLMPC and CNLMPC

This section will compare the performances between HNLMPC and existing CNLMPC, including economic performance and computational power requirements. Compared with CNLMPC, HNLMPC has a larger sampling time and is unable to capture more precise variations in disturbance parameters. A computer with Intel(R) Core (TM) i5-10400 F (2.9 GHz) CPU and 24 GB memory is used for simulations.

Figure 8 (a) illustrates the performance of HNLMPC and CNLMPC compared to the Basic R-b control method. Electricity costs vary with changes in the prediction horizon. With an increase in the prediction horizon, HNLMPC's performance improves, while CNLMPC's initially decreases before rebounding. It can be inferred that increasing the prediction horizon has both advantages and disadvantages on MPC performance. As the prediction horizon expands, it enhances the system's long-term planning and responsiveness, resulting in improved cost-effectiveness. This is the primary reason for the upward trend observed in CNPMPC's performance curve in the latter part. However, the accumulation of prediction errors and local optimization challenges associated with solving non-convex optimization problems degrade the performance of nonlinear MPC, contributing to the downward trend observed in CNPMPC's performance curve in the earlier part. Despite HNLMPC's longer sampling time, which reduces the controller's dynamic response capability, its enhanced optimization solving capability leads to consistently higher energy savings, cost-effectiveness, and peak energy reduction compared to CNLMPC under the same prediction horizon.

Figure 8 (b) shows the average simulation time for a single sampling time. Compared with CNLMPC, HNLMPC consumes less simulation time and requires fewer computational resources. HNLMPC is characterized by high-cost savings and peak load shifting performances, with lower computational power requirements, making it well-suited for the residential inverter AC to realize advanced DR control, especially for the massive installed ACs.

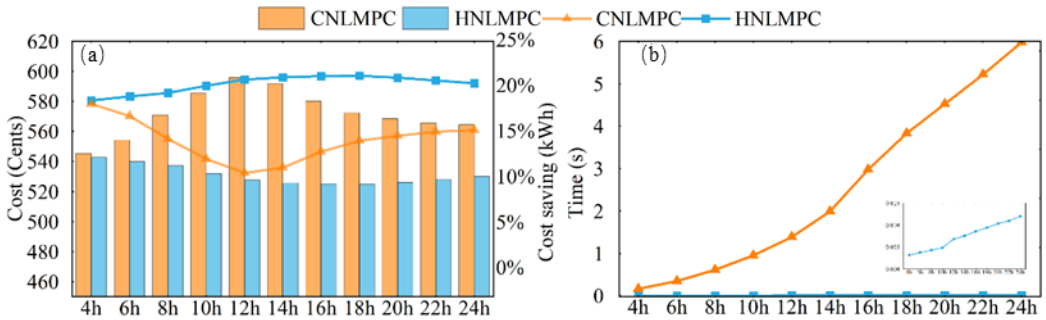

Figure 8: Performance comparison with the variations of prediction horizons between HNLMPC and CNLMPC during the typical days including (a) energy consumption; (b) peak energy consumption; (c) electricity cost and (d) simulation time

5. Conclusion

In this study, a novel hierarchical model predictive control framework was tailored for inverter ACs to avoid challenging and complex control system integration processes in applying MPC for inverter air conditioners. To improve energy efficiency and comfort, a comprehensive dynamic nonlinear model integrating the dynamic nonlinear performances of inverter AC and the building was developed to achieve continuously variable frequency optimization operation of the cooling system. Comparisons were made with various control strategies to thoroughly assess the effectiveness of the proposed approach by simulation. The findings of this study held the potential to enhance the performance of MPC and facilitate its application in real-world buildings, contributing to carbon neutrality policies. The main conclusions are as follows:

- Compared with the basic rule-based controller, HNLMPC could reduce peak-hour energy consumption by 31.6% and the total electricity cost by 14.3 % over the entire cooling season.
- The developed HMPC had less dependency on model accuracy and always ensured thermal comfort in the building, even in the presence of significant model errors.
- When the prediction model considered the dynamic characteristics of the inverter AC under different disturbances and interactions with the building, the HNLMPC could achieve more DR and cost savings with minimal energy sacrifice.
- Compared with CMPC, the developed HMPC could realize higher demand response efficiency and require less calculation power.

NOMENCLATURE

CMPC	centralized MPC	(-)
COP	coefficient of performance	(W/W)
Cost	cost of electricity	(Cent)
DR	demand response	(-)
EP	electricity price	(-)
HNLMPC	hierarchical nonlinear MPC	(-)
IoT	Internet of Things	(-)
MPC	Model predictive control	(-)
N	prediction horizons	(1)
P	power	(W)

RBC	rule-based control	(-)
RTP	real-time pricing	(Cent/kWh)
STD	standard deviation	(-)
$\triangle\tau$	sampling time	(s)

Subscript

ref	reference value
in	indoor air
up	upper layer
low	lower layer

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